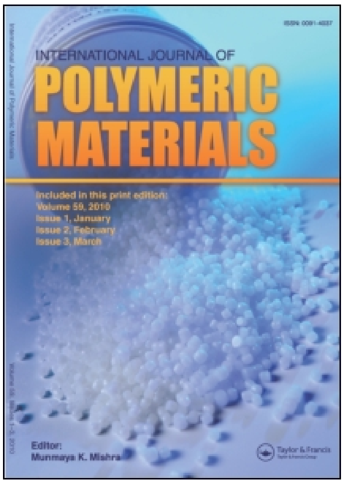

International Journal of Polymeric Materials
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713647664

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To cite this Article Ito, Katsuhiko , Arimatsu, Ikuro and Wang, J. H.(1982) 'Broken Section Method for Analyzing Non-Newtonian Flow of Polymer Melts in Wire Coating Extrusion Die', International Journal of Polymeric Materials, 9: 2, 125 — 138
To link to this Article: DOI: 10.1080/00914038208077973
URL: http://dx.doi.org/10.1080/00914038208077973

Intern. J. Polymeric Mater., 1982, Vol. 9, pp. 125–138
0091-4037/82/0901-0125 $06.50/0

Printed in Great Britain

Broken Section Method for Analyzing Non-Newtonian Flow of Polymer Melts in Wire Coating Extrusion Die

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(*Received May 6, 1981*)

It is proposed in this paper that non-Newtonian flow behavior of polymer melts in wire coating extrusion die can be analyzed theoretically by broken section method. It is assumed that the flow is steady, isothermal and laminar and the polymer melts behave as a power law fluid with flow index n. Further we assume that there occurs no slip at the boundaries. The polymer melts filled between die inside and wire surface are broken perpendicular to the flow axis into many sections, each of unit length. The materials in each section having different heights with the variation of the diameters of die, behave under combined pressure and drag flows of power law fluid. In this problem, the simultaneous equations on equal volumetric flow rate through each section can be formulated and hence the pressure distribution in the direction of wire axis can be obtained, for four cases where the relationships between drag flow and pressure flow are varied. The mathematical results are presented in detail for the cases of Newtonian flow ($n = 1$) and pseudoplastic flow ($n = 1/2$).

I. FUNDAMENTALS

Broken section method[1–4] for analyzing successfully boundary value problems on polymer melt processing, with particular emphasis on extrusion die design problems, is extended to solve theoretically melt flow problem on wire coating extrusion.[5–7]

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† All communications should be addressed.

This report is one of the additional applications of the broken section method.

Wire (radius R_i) is travelled with linear velocity V into the coaxial die (inner radius R_0) filled by molten polymers. The flow problem[8,9] in wire coating extrusion simulates the combined pressure and drag flows.[10,11] In this problem, no slip at all the stationary and moving boundaries is assumed.[12,13] In addition, polymer melts are to be incompressive and both the gravitational and inertia effects are negligible.

It is assumed further that the flow is isothermal and laminar and that the power law in shear flow is applicable.

The power law [14,15] is written in the form

$$\tau = \eta\dot{\gamma} \tag{I-1}$$

$$\eta = \eta^0\left(\frac{\dot{\gamma}}{\dot{\gamma}^0}\right)^{n-1} \tag{I-2}$$

where τ is shear stress and η is the viscosity at shear rate $\dot{\gamma}$. The flow index of the fluid is n and η_0 is the viscosity at the standard shear rate $\dot{\gamma}^0$.

For the purpose of the analysis, the extrusion die is broken perpendicularly to the Z-axis into N sections, each of length S, as shown in Figure 1. The pressure at the start of the jth section is p_{j-1} and at the end is p_j.

Hence $(p_j - p_{j-1})/S$ is the pressure gradient of the jth section. As shown in Figure 2, the polymer melts in each section having different heights in accordance with the variation of the inner diameters of die, behave under

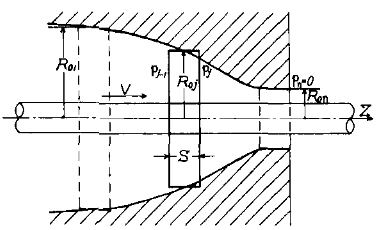

FIGURE 1 Notation for analysis of the flow of polymer melts in wire coating extrusion die.

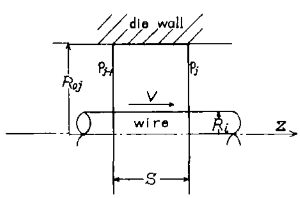

FIGURE 2 The jth section of polymer melts being wire drawn and showing principal dimension.

combined pressure and drag flows of power law fluid. Depending upon the mutual relationship between pressure and drag flow, the geometrical configuration of die,[8] the wire travelling velocity V and further the flow behaviors of polymer melts, in general, the flow problem of the jth section in wire coating extrusion die is summarized and analyzed in the following four cases.

II. BROKEN SECTION METHOD

(A). The case of Figure 3(a)

Both pressure gradient and velocity gradient are negative.

Introducing the following dimensionless parameters,

$$\rho \equiv r/R_i \tag{II–1}$$

$$\phi \equiv v_z/\Gamma V \tag{II–2}$$

$$\Gamma \equiv \frac{R_i}{V}\left[R_i\left|\frac{p_j - p_{j-1}}{S}\right|\frac{(\dot{\gamma}^0)^{n-1}}{\eta^0}\right]^{1/n} \tag{II–3}$$

where r: radial distance in cylindrical coordinate
Z: axial distance in cylindrical coordinate
v_z: velocity component in Z-axis

the Z component of the momentum equation on wire coating flow problem becomes

$$\frac{d}{d\rho}\left[\rho\left(-\frac{d\phi}{d\rho}\right)^n\right] = \rho \tag{II–4}$$

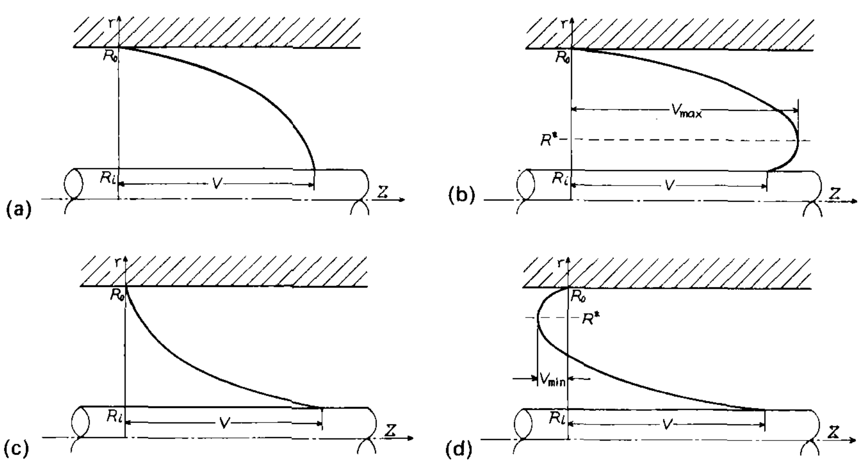

FIGURE 3 Four types of velocity profiles in the flow of polymer melts in die.

with the boundary conditions:

$$r = R_0: \quad \phi = 0 \tag{II–5}$$

$$r = R_i: \quad \phi = \Gamma^{-1} \tag{II–6}$$

(B). The case of Figure 3(b)

Pressure gradient is negative and velocity profile has maximum.

In this case, the velocity profile has the maximum at $r = R^*$. Hence the analysis should be performed in the following two domains.

(1) *Inner domain*: $R_i < r < R^*$

$$\frac{d}{d\rho}\left[\rho\left(\frac{d\phi_i}{d\rho}\right)^n\right] = -\rho \tag{II–7}$$

where ϕ_i: ϕ for the inner domain.

(2) *Outer domain*: $R^* < r < R_0$

$$\frac{d}{d\rho}\left[\rho\left(-\frac{d\phi_0}{d\rho}\right)^n\right] = \rho \tag{II–8}$$

where ϕ_0: ϕ for the outer domain.

The boundary conditions are as follows.

$$r = R_i: \quad \phi_i = \Gamma^{-1} \tag{II–9}$$

$$r = R_0: \quad \phi_0 = 0 \tag{II–10}$$

$$r = R^*: \quad d\phi_0/d\rho = 0 \tag{II–11}$$

$$r = R^*: \quad d\phi_i/d\rho = 0 \tag{II–12}$$

(C). The case of Figure 3 (c)

Pressure gradient is positive but velocity gradient is negative.

In this case, the momentum equation is

$$\frac{d}{d\rho}\left[\rho\left(-\frac{d\phi}{d\rho}\right)^n\right] = -\rho \tag{II–13}$$

with the boundary conditions:

$$r = R_0: \quad \phi = 0 \tag{II–14}$$

$$r = R_i: \quad \phi = \Gamma^{-1} \tag{II–15}$$

(D). The case of Figure 3(d)

Pressure gradient is positive and velocity profile has minimum.

In this case, the velocity profile has the minimum at $r = R^*$. Hence the analysis should be done in the following two domains.

(1) *Inner domain*: $R_i < r < R^*$

$$\frac{d}{d\rho}\left[\rho\left(-\frac{d\phi_i}{d\rho}\right)^n\right] = -\rho \tag{II–16}$$

where ϕ_i: ϕ for the inner domain.

(2) *Outer domain*: $R^* < r < R_0$

$$\frac{d}{d\rho}\left[\rho\left(\frac{d\phi_0}{d\rho}\right)^n\right] = \rho \tag{II–17}$$

where ϕ_0: ϕ for the outer domain.

The boundary conditions are as follows.

$$r = R_i: \quad \phi_i = \Gamma^{-1} \tag{II–18}$$

$$r = R_0: \quad \phi_0 = 0 \tag{II–19}$$

$$r = R^*: \quad d\phi_i/d\rho = 0 \tag{II–20}$$

$$r = R^*: \quad d\phi_0/d\rho = 0 \tag{II–21}$$

In the case where the mutual relationship between pressure and drag flows, the geometrical profile of die, the wire travelling velocity V and further the flow behaviors of polymer melts are known, the velocity distribution of (II–4), (II–7), (II–8), (II–13), (II–16) and (II–17) can be solved.

Hence the volumetric flow rate Q through each broken section can be obtained from the integral of the velocity profile. Because the volumetric flow rate through each broken section is always equal to the constant Q and the exit pressure should be zero, the simultaneous equation on the equal volumetric flow rate Q through each broken section contains N unknown p_j's and hence the flow problem in wire coating extrusion die is formulated completely.

III. NEWTONIAN CASE ($n = 1$)

As one of the easiest examples, the analytical results for newtonian case ($n = 1$) are shown, as follows, in the same order of the previous four cases: (A), (B), (C) and (D).

(A). Figure 3(a), $(p_j - p_{j-1})/S < 0$

The differential equation (A) is as follows.

$$\frac{d}{d\rho}\left[\rho\left(-\frac{d\phi}{d\rho}\right)\right] = \rho \tag{III-1}$$

The analytical result is obtained as

$$\phi = -\frac{\rho^2}{4} - C_1 \ln|\rho| + C_2 \tag{III-2}$$

where C_1 and C_2 are integration constants and can be determined by the following boundary conditions.

$$r = R_{0j}, \quad \text{i.e. } \phi = 0 \tag{III-3}$$

$$r = R_i, \quad \text{i.e. } \phi = \Gamma^{-1} \tag{III-4}$$

Namely

$$C_1 = \frac{4\Gamma^{-1} + 1 - \beta_j^2}{4 \ln \beta_j} \tag{III-5}$$

$$C_2 = \Gamma^{-1} + \frac{1}{4} \tag{III-6}$$

where

$$\beta_j \equiv \frac{R_{0j}}{R_i} \tag{III-7}$$

The volumetric flow rate Q can be expressed as

$$Q = \frac{2\pi R_i^4}{\eta^0}\left|\frac{p_j - p_{j-1}}{S}\right| \int_1^{\beta_j} \phi\rho\, d\rho$$

$$= \frac{2\pi R_i^4}{\eta^0}\left|\frac{p_j - p_{j-1}}{S}\right| \left\{\frac{4\Gamma^{-1} + 1 - \beta_j^2}{16 \ln \beta_j}(\beta_j^2 - 1) + \frac{1}{16}(\beta_j^4 - 1) - \frac{\Gamma^{-1}}{2}\right\} \tag{III-8}$$

Γ, C_1 and C_2 are a function of $(p_j - p_{j-1})/S$.

(B). Figure 3(b), $(p_j - p_{j-1})/S < 0$

The velocity profile for (B) has an extreme value somewhere between the die wall and the surface of the wire, at $r = R^*$.

It is necessary, then, to write the differential equation for each region separately and account for the proper sign for the shear rate in the absolute value sign. In a similar way, Eq. (II–7) in the lower part of the velocity profile, where $d\phi_i/d\rho > 0$ is as follows.

$$\frac{d}{d\rho}\left[\rho\left(\frac{d\phi_i}{d\rho}\right)\right] = -\rho \tag{III–9}$$

In the upper part of the velocity profile, where $d\phi_0/d\rho < 0$, on the other hand, Eq. (II–8) can be written as

$$\frac{d}{d\rho}\left[\rho\left(-\frac{d\phi_0}{d\rho}\right)\right] = \rho \tag{III–10}$$

The boundary conditions for this case are as follows.

$$r = R_i: \qquad \phi_i = \Gamma^{-1} \tag{III–11}$$

$$r = R_{0j}: \qquad \phi_0 = 0 \tag{III–12}$$

$$r = R^*: \quad d\phi_i/d\rho = 0 \tag{III–13}$$

$$r = R^*: \quad d\phi_0/d\rho = 0 \tag{III–14}$$

The solutions are

$$\phi_i = -\frac{\rho^2}{4} + \frac{\beta_j^{*2}}{2}\ln|\rho| + \Gamma^{-1} + \frac{1}{4} \tag{III–15}$$

$$\phi_0 = -\frac{\rho^2}{4} + \frac{\beta_j^{*2}}{2}\ln|\rho| + \frac{\beta_j^2}{4} - \frac{\beta_j^{*2}}{2}\ln\beta_j \tag{III–16}$$

where

$$\beta_j^* \equiv R^*/R_i \tag{III–17}$$

The boundary condition for β_j^* to be determined is as follows.

$$r = R^*: \quad \phi_i = \phi_0 \tag{III–18}$$

The value of β_j^* can be determined by the following equation obtained from the boundary condition (III–18).

$$\Gamma^{-1} + \frac{1}{4} = \frac{\beta_j^2}{4} - \frac{\beta_j^{*2}}{2}\ln\beta_j \tag{III–19}$$

The volumetric flow rate Q is obtained by integrating the velocity equation.

$$Q = \frac{2\pi R_i^4}{\eta^0}\left|\frac{p_j - p_{j-1}}{S}\right|\left\{\int_1^{\beta_j^*}\phi_i\rho\, d\rho + \int_{\beta_j^*}^{\beta_j}\phi_0\rho\, d\rho\right\} \tag{III–20}$$

$$\int_1^{\beta_j^*}\phi_i\rho\, d\rho = \left(\frac{\beta_j^*}{2}\right)(4\ln\beta_j^* - 3) + \left(\frac{\beta_j^*}{2}\right)^2(1 + 2\Gamma^{-1}) - \frac{\Gamma^{-1}}{2} - \frac{1}{16} \tag{III–21}$$

$$\int_{\beta_j^*}^{\beta_j} \phi_0 \rho \, d\rho = \frac{1}{16}(\beta_j^2 - 3\beta_j^{*2})(\beta_j^2 - \beta_j^{*2}) + \frac{\beta_j^{*4}}{4}(\ln \beta_j - \ln \beta_j^*) \tag{III-22}$$

Γ and β_j^* are a function of $(p_j - p_{j-1})/S$.

(C). Figure 3(c). $(p_j - p_{j-1})/S > 0$

The differential equation for (C) is as follows.

$$\frac{d}{d\rho}\left[\rho\left(-\frac{d\phi}{d\rho}\right)\right] = -\rho \tag{III-23}$$

The analytical result can be written as

$$\phi = \frac{\rho^2}{4} - C_3 \ln|\rho| + C_4 \tag{III-24}$$

where C_3 and C_4 are integration constants and can be determined by the following boundary conditions.

$$r = R_{0j}, \quad \text{i.e.} \quad \phi = 0 \tag{III-25}$$

$$r = R_i, \quad \text{i.e.} \quad \phi = \Gamma^{-1} \tag{III-26}$$

In the similar way to (A), C_3 and C_4 can be determined as

$$C_3 = \frac{\beta_j^2 + 4\Gamma^{-1} - 1}{4 \ln \beta_j} \tag{III-27}$$

$$C_4 = \Gamma^{-1} - \frac{1}{4} \tag{III-28}$$

The volumetric flow rate Q can be expressed as

$$Q = \frac{2\pi R_i^4}{\eta^0}\left|\frac{p_j - p_{j-1}}{S}\right| \int_1^{\beta_j} \phi \rho \, d\rho$$

$$= \frac{2\pi R_i^4}{\eta^0}\left|\frac{p_j - p_{j-1}}{S}\right| \left\{\frac{\beta_j^2 + 4\Gamma^{-1} - 1}{16 \ln \beta_j}(\beta_j^2 - 1) - \frac{1}{16}(\beta_j^4 - 1) - \frac{\Gamma^{-1}}{2}\right\} \tag{III-29}$$

Γ, C_3 and C_4 are a function of $(p_j - p_{j-1})/S$.

(D). Figure 3(d). $(p_j - p_{j-1})/S > 0$

The velocity profile for (D) has an extreme value somewhere between die wall and wire surface, at $r = R^*$. In a similar way to (B), hence, it is necessary to write the differential equation for each region separately and account for the proper sign for the shear rate in the absolute value sign.

$$\frac{d}{d\rho}\left[\rho\left(-\frac{d\phi_i}{d\rho}\right)\right] = -\rho \tag{III-30}$$

$$\frac{d}{d\rho}\left[\rho\left(\frac{d\phi_0}{d\rho}\right)\right] = \rho \tag{III-31}$$

The boundary conditions for (D) can be written as

$$r = R_i: \qquad \phi = \Gamma^{-1} \tag{III-32}$$

$$r = R_{0j}: \qquad \phi = 0 \tag{III-33}$$

$$r = R^*: \quad d\phi_i/d\rho = 0 \tag{III-34}$$

$$r = R^*: \quad d\phi_0/d\rho = 0 \tag{III-35}$$

The solutions are

$$\phi_i = \frac{\rho^2}{4} - \frac{\beta_j^{*2}}{2} \ln|\rho| + \Gamma^{-1} - \frac{1}{4} \tag{III-36}$$

$$\phi_0 = \frac{\rho^2}{4} - \frac{\beta_j^{*2}}{2} \ln|\rho| + \frac{\beta_j^{*2}}{2} \ln \beta_j - \frac{\beta_j^2}{4} \tag{III-37}$$

where the boundary condition for β_j^* to be determined is as follows.

$$r = R^*: \quad \phi_i = \phi_0 \tag{III-38}$$

The value of β_j^* can be determined by the following equation obtained from the boundary condition (III–38).

$$\Gamma^{-1} - \frac{1}{4} = \frac{\beta_j^{*2}}{2} \ln \beta_j - \frac{\beta_j^2}{4} \tag{III-39}$$

The volumetric flow rate Q is obtained by integrating the velocity equation.

$$Q = \frac{2\pi R_i^4}{\eta^0}\left|\frac{p_j - p_{j-1}}{S}\right|\left[\int_1^{\beta_j^*} \phi_i \rho \, d\rho + \int_{\beta_j^*}^{\beta_j} \phi_0 \rho \, d\rho\right] \tag{III-40}$$

$$\int_1^{\beta_j^*} \phi_i \rho \, d\rho = \left(\frac{\beta_j^*}{2}\right)^4 (3 - 4 \ln \beta_j^*) + \left(\frac{\beta_j^*}{2}\right)^2 (2\Gamma^{-1} - 1) - \frac{\Gamma^{-1}}{2} + \frac{1}{16} \tag{III-41}$$

$$\int_1^{\beta_j^*} \phi_0 \rho \, d\rho = -\frac{1}{16}(\beta_j^2 - 3\beta_j^{*2})(\beta_j^2 - \beta_j^{*2}) + \frac{\beta_j^{*4}}{4}(\ln \beta_j^* - \ln \beta_j) \tag{III-42}$$

Γ and β_j^* are a function of $(p_j - p_{j-1})/S$.

IV. PSEUDOPLASTIC CASE ($n = 1/2$)

As one of the typical examples of non-Newtonian case, the analytical results for ($n = 1/2$) are shown, as follows, in the same order of the previous four cases: (A), (B), (C) and (D).

(A). Figure 3(a), $(p_j - p_{j-1})/S < 0$

The differential equation for (A) is as follows.

$$\frac{d}{d\rho}\left[\rho\left(-\frac{d\phi}{d\rho}\right)^{1/2}\right] = \rho \tag{IV-1}$$

The analytical result can be written as

$$\phi = -\rho^3/12 - C_5\rho + C_5^2/\rho + C_6 \tag{IV-2}$$

where C_5 and C_6 are integration constants and can be determined by the following boundary conditions.

$$r = R_{0j}, \quad \text{i.e.} \quad \phi = 0 \tag{IV-3}$$

$$r = R_i, \quad \text{i.e.} \quad \phi = \Gamma^{-1} \tag{IV-4}$$

Namely

$$C_5 = -\frac{\beta_j}{2} + \sqrt{-\frac{1}{12}\beta_j(\beta_j - 1)^2 - \frac{\beta_j}{\Gamma(\beta_j - 1)}} \tag{IV-5}$$

$$C_6 = \frac{1}{12}(\beta_j + 1)(\beta_j^2 - 6\beta_j + 1) + \frac{1}{\Gamma(\beta_j - 1)} + (\beta_j + 1) \times \sqrt{-\frac{1}{12}\beta_j(\beta_j - 1)^2 - \frac{\beta_j}{\Gamma(\beta_j - 1)}} \tag{IV-6}$$

The volumetric flow rate Q can be expressed as

$$\begin{aligned} Q &= 2\pi R_i^3\left[R_i\frac{(\dot{\gamma}^0)^{-1/2}}{\eta^0}\right]^2\left|\frac{p_j - p_{j-1}}{S}\right|^2\int_1^{\beta_j}\phi\rho\,d\rho \\ &= 2\pi R_i^3\left[R_i\frac{(\dot{\gamma}^0)^{-1/2}}{\eta^0}\right]^2\left|\frac{p_j - p_{j-1}}{S}\right|^2 \\ &\quad\times\left\{\frac{(\beta_j - 1)^3}{6}\sqrt{-\frac{1}{12}\beta_j(\beta_j - 1)^2 - \frac{\Gamma^{-1}\beta_j}{\beta_j - 1}} + \frac{(\beta_j - 1)^5}{40} - \frac{\Gamma^{-1}(\beta_j - 1)}{2}\right\} \end{aligned} \tag{IV-7}$$

Γ, C_5 and C_6 are a function of $(p_j - p_{j-1})/S$.

(B). Figure 3(b). $(p_j - p_{j-1})/S < 0$

The velocity profile for (B) has an extreme value somewhere between the die wall and the surface of the wire, at $r = R^*$.

In a similar way to III-(B), the differential equation for each region separately can be written as

$$\frac{d}{d\rho}\left[\rho\left(\frac{d\phi_i}{d\rho}\right)^{1/2}\right] = -\rho \tag{IV–8}$$

$$\frac{d}{d\rho}\left[\rho\left(-\frac{d\phi_0}{d\rho}\right)^{1/2}\right] = \rho \tag{IV–9}$$

The boundary conditions for (B) can be written as

$$r = R_i: \qquad \phi_i = \Gamma^{-1} \tag{IV–10}$$

$$r = R_{0j}: \qquad \phi_0 = 0 \tag{IV–11}$$

$$r = R^*: \quad d\phi_i/d\rho = 0 \tag{IV–12}$$

$$r = R^*: \quad d\phi_0/d\rho = 0 \tag{IV–13}$$

The solutions are

$$\phi_i = \frac{\rho^3}{12} - \frac{\beta_j^{*2}}{2}\rho - \frac{\beta_j^{*4}}{4\rho} + \Gamma^{-1} + \frac{3\beta_j^{*4} + 6\beta_j^{*2} - 1}{12} \tag{IV–14}$$

$$\phi_0 = -\frac{\rho^3}{12} + \frac{\beta_j^{*2}}{2}\rho + \frac{\beta_j^{*4}}{4\rho} + \frac{\beta_j^4 - 6\beta_j^{*2}\beta_j^2 - 3\beta_j^{*4}}{12\beta} \tag{IV–15}$$

where the boundary condition for β_j^* to be determined is as follows.

$$r = R^*: \quad \phi_i = \phi_0 \tag{IV–16}$$

The value of β_j^* can be determined by the following equation obtained from the boundary condition (IV–16).

$$\frac{4\beta_j^{*3}}{3} + \frac{\beta_j^4 - 6\beta_j^{*2}\beta_j^2 - 3\beta_j^{*4}}{12\beta} = \Gamma^{-1} + \frac{3\beta_j^{*4} + 6\beta_j^{*2} - 1}{12} \tag{IV–17}$$

The volumetric flow rate Q is obtained by integrating the velocity equation.

$$Q = 2\pi R_i^3\left[R_i\frac{(\dot{\gamma}^0)^{1/2}}{\eta^0}\right]^2\left|\frac{p_j - p_{j-1}}{S}\right|^2\left\{\int_1^{\beta_j^*}\phi_i\rho\, d\rho + \int_{\beta_j^*}^{\beta_j}\phi_0\rho\, d\rho\right\} \tag{IV–18}$$

$$\int_1^{\beta_j^*}\phi_i\rho\, d\rho = \frac{(\beta_j^* - 1)^4(5\beta_j^{*2} + 4\beta_j^* + 1)}{40} + \frac{\Gamma^{-1}}{2}(\beta_j^{*2} - 1) \tag{IV–19}$$

$$\int_{\beta_j^*}^{\beta_j} \phi_0 \rho \, d\rho = \frac{1}{8\beta_j}(\beta_j^* - \beta_j)(\beta_j^* + \beta_j)(\beta_j^{*2} + \beta_j^2)(\beta_j^{*2} + 3\beta_j^2) \tag{IV-20}$$

Γ and β_j^* are a function of $(p_j - p_{j-1})/S$.

(C). Figure 3(c), $(p_j - p_{j-1})/S > 0$

The differential equation for (C) is as follows.

$$\frac{d}{d\rho}\left[\rho\left(-\frac{d\phi}{d\rho}\right)^{1/2}\right] = -\rho \tag{IV-21}$$

The analytical result can be written as

$$\phi = -\frac{\rho^3}{12} + C_7 + \frac{C_7^2}{\rho} + C_8 \tag{IV-22}$$

where C_7 and C_8 are integration constants and can be determined by the following boundary conditions.

$$r = R_{0j}, \quad \text{i.e.} \quad \phi = 0 \tag{IV-23}$$

$$r = R_i, \quad \text{i.e.} \quad \phi = \Gamma^{-1} \tag{IV-24}$$

In the similar to III-(C), C_7 and C_8 are determined as

$$C_7 = \frac{\beta_j}{2} + \sqrt{-\frac{1}{12}\beta_j(\beta_j - 1)^2 + \frac{\beta_j}{\Gamma(\beta_j - 1)}} \tag{IV-25}$$

$$C_8 = \frac{1}{12}(\beta_j + 1)(\beta_j^2 - 6\beta_j + 1) - \frac{1}{\Gamma(\beta_j - 1)} - (\beta_j + 1) \times \sqrt{-\frac{1}{12}\beta_j(\beta_j - 1)^2 + \frac{\beta_j}{\Gamma(\beta_j - 1)}} \tag{IV-26}$$

The volumetric flow rate Q can be expressed as

$$\begin{aligned} Q &= 2\pi R_i^3 \left[R_i \frac{(\dot{\gamma}^0)^{-1/2}}{\eta^0}\right]^2 \left|\frac{p_j - p_{j-1}}{S}\right|^2 \int_1^{\beta_j} \phi\rho \, d\rho \\ &= 2\pi R_i^3 \left[R_i \frac{(\dot{\gamma}^0)^{-1/2}}{\eta_0}\right]^2 \left|\frac{p_j - p_{j-1}}{S}\right|^2 \\ &\quad \times \left\{-\frac{(\beta_j - 1)^3}{6}\sqrt{-\frac{1}{12}\beta_j(\beta_{j-1})^2 - \frac{\Gamma^{-1}\beta_j}{\beta_j - 1}} + \frac{(\beta_j - 1)^5}{40} + \frac{\Gamma^{-1}(\beta_j - 1)}{2}\right\} \end{aligned} \tag{IV-27}$$

Γ, C_7 and C_8 are a function of $(p_j - p_{j-1})/S$.

(D). Figure 3(d). $(p_j - p_{j-1})/S > 0$

The velocity profile for (D) has an extreme value somewhere between the die wall and the surface of the wire, at $r = R^*$.

In a similar way to III-(D), the differential equation for each region separately can be written as

$$\frac{d}{d\rho}\left[\rho\left(-\frac{d\phi_i}{d\rho}\right)^{1/2}\right] = -\rho \tag{IV–28}$$

$$\frac{d}{dp}\left[\rho\left(\frac{d\phi_0}{d\rho}\right)^{1/2}\right] = \rho \tag{IV–29}$$

The boundary conditions for (D) can be written as

$$r = R_i: \qquad \phi_i = \Gamma^{-1} \tag{IV–30}$$

$$r = R_{0j}: \qquad \phi_0 = 0 \tag{IV–31}$$

$$r = R^*: \quad d\phi_i/d\rho = 0 \tag{IV–32}$$

$$r = R^*: \quad d\phi_0/d\rho = 0 \tag{IV–33}$$

The solutions are

$$\phi_i = -\frac{\rho^3}{12} + \frac{\beta_j^{*2}}{2}\rho + \frac{\beta_j^{*4}}{4\rho} + \Gamma^{-1} - \frac{3\beta_j^{*4} + 6\beta_j^{*2} - 1}{12} \tag{IV–34}$$

$$\phi_0 = \frac{\rho^3}{12} - \frac{\beta_j^{*2}}{2}\rho - \frac{\beta_j^{*4}}{4\rho} + \frac{3\beta_j^{*4} + 6\beta_j^{*2}\beta_j^2 - \beta_j^4}{12\beta_j} \tag{IV–35}$$

where the boundary condition for β_j^* to be determined is as follows.

$$r = R^*: \quad \phi_i = \phi_0 \tag{IV–36}$$

The value of β_j^* can be determined by the following equation obtained from the boundary condition (IV–36).

$$\frac{4\beta_j^{*3}}{3} + \Gamma^{-1} - \frac{3\beta_j^{*4} + 6\beta_j^{*2} - 1}{12} = \frac{3\beta_j^{*4} + 6\beta_j^{*2}\beta_j^2 - \beta_j^4}{12\beta_j} \tag{IV–37}$$

The volumetric flow rate Q is obtained by integrating the velocity equation.

$$Q = 2\pi R_i^3\left[R_i\frac{(\dot{\gamma}^0)^{-1/2}}{\eta_0}\right]^2\left|\frac{p_j - p_{j-1}}{S}\right|^2\left\{\int_1^{\beta_j^*}\phi_i\rho\, d\rho + \int_{\beta_j^*}^{\beta_j}\phi_0\rho\, d\rho\right\} \tag{IV–38}$$

$$\int_1^{\beta_j^*}\phi_i\rho\, d\rho = -\frac{(\beta_j^* - 1)^4(5\beta_j^{*2} + 4\beta_j^* + 1)}{40} + \frac{\Gamma^{-1}}{2}(\beta_j^{*2} - 1) \tag{IV–39}$$

$$\int_{\beta_j^*}^{\beta_j}\phi_0\rho\, d\rho = -\frac{1}{8\beta_j}(\beta_j^* - \beta_j)(\beta_j^* + \beta_j)(\beta_j^{*2} + \beta_j^2)(\beta_j^{*2} + 3\beta_j^2) \tag{IV–40}$$

Γ and β_j^* are a function of $(p_j - p_{j-1})/S$.

Acknowledgement

This paper is one additional work of our numerous researches,[2] entitled "Broken Section Method for Analyzing Boundary Value Problems on Molten Flow in Polymer Processing—With Particular Emphasis on Extrusion Die Design—", which was originated with our first paper[1] with Dean J. M. McKelvey, Washington University in St. Louis (Missouri, USA), during the appointment of one of the authors: Professor K. Ito at Washington University in St. Louis for one year (1969–1970) under the auspices of the Senior Foreign Scientist Fellowship of the US National Science Foundation. The authors express their appreciation to Dean J. M. McKelvey for his initial suggestion of the problem.

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